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CONVERSION IN A TWO-REGION REACTOR

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February 16, 1953

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Work performed under Contract No. W-7405-Eng-26.

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OAK RIDGE NATIONAL LABORATORY
Operated By
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POST OFFICE BOX P
OAK RIDGE, TENNESSEE

Table of Contents

Summary

Calculations were performed by two group methods to determine the critical concentration, neutron balance and conversion ratio in a two-region reactor. The reactor had the following geometry: a four foot diameter spherical core of uranyl sulfate dissolved in heavy water is contained in a shell of stainless steel surrounded by a two foot thick spherical blanket containing thorium and heavy water. Two stainless steel core shells, 1/8 inch thick and 1/4 inch thick were considered. For each core shell, three concentrations of thorium were supposed: 1000 g thorium per liter (as a slurry of thorium oxide), 4218 grams thorium per liter (as pellets of thorium oxide), and 7000 grams of thorium per liter (as spheres of thorium metal). For calculation purposes the thorium distribution was considered homogeneous in all cases. Further, it was assumed that the reactor was free of poisons and that no multiplication occurred in the blanket. Thus, the only materials considered present were uranyl sulfate, heavy water, stainless steel, and thorium (as oxide or metal). A complete tabulation of results is to be found on page 15.

Theory of the Calculations

1. Introduction

The calculations were performed using the methods and nomenclature of S. Visner (CF-51-10-110). Since the process by which the calculation sheets displayed on pp. 51 ff. of that report were obtained is discussed in outline only, the authors of this memorandum considered it useful to provide readers with a more extensive treatment. Much of the material presented here was obtained by private communication with Dr. Visner. Considerable reference was made to "The Elements of Nuclear Reactor Theory," by Glasstone and Edlund, Chapter VIII.

The procedure used was the following: the differential equations for the fast and slow fluxes in the core and blanket were written. The boundary conditions used were: 1) continuity of fast and slow fluxes at the interface between core and blanket; 2) vanishing of fast and slow fluxes at the outside edge of the reactor; 3) continuity of the blanket and core fast currents at the interface; 4) a balance of slow currents at the blanket-core interface to include shell absorption; and 5) finite, non-negative fluxes exist everywhere in the reactor. Given the reactor dimensions, the necessary cross sections, and core and blanket compositions, it is possible to calculate the permissible thickness of the shell which will allow criticality. The flux distributions were obtained in terms of the radius variable and a multiplicative factor A, whose magnitude would depend upon the reactor power. We then can calculate a neutron balance based upon one neutron being absorbed in an atom of U^{235}. This balance gives us the conversion ratio (atoms of U^{233} produced per atom of U^{235} destroyed) as well as fast and slow leakages and absorptions. While no calculations were made to include effects of poisons and blanket multiplication, it is possible to consider such factors given the information contained here.

2. <u>Derivations</u> (See table of nomenclature, p. 16)

A. <u>Core equations</u>

The usual two group equations are written for the core:

$$-D_{FC} \nabla^2 \phi_{FC} - k_c \Sigma_{sc} \phi_{sc} + \Sigma_{FC} \phi_{FC} = 0 \qquad (1)$$

$$-D_{sc} \nabla^2 \phi_{sc} + \Sigma_{sc} \phi_{sc} - \Sigma_{FC} \phi_{FC} = 0 \qquad (2)$$

The assumption is made here that the resonance escape probability for the core is unity, since the core solutions have a very high moderator-to-fuel ratio.

We note that if we eliminate ϕ_{FC} between equations (1) and (2) we obtain the equation:

$$\frac{D_{sc} D_{FC}}{k_c \Sigma_{sc}} \nabla^4 \phi_{sc} - \frac{1}{k_c} \left[\frac{D_{sc} \Sigma_{FC}}{\Sigma_{sc}} + D_{FC}\right] \nabla^2 \phi_{sc} + \left[\frac{1}{k_c} - 1\right] \Sigma_{FC} \phi_{sc} = 0 \quad (3)$$

If we eliminate ϕ_{sc} between (1) and (2) we obtain:

$$\frac{D_{sc} D_{FC}}{k_c \Sigma_{sc}} \nabla^4 \phi_{FC} - \frac{1}{k_c} \left[\frac{D_{sc} \Sigma_{FC}}{\Sigma_{sc}} + D_{FC}\right] \nabla^2 \phi_{FC} + \left[\frac{1}{k_c} - 1\right] \Sigma_{FC} \phi_{FC} = 0 \quad (4)$$

The coefficients in (3) are identical with those in (4). We may rewrite (3) and (4) as follows:

$$\nabla^4 \phi_{sc} - \left[B_{sc}^2 + B_{FC}^2\right] \nabla^2 \phi_{sc} + \left[1 - k_c\right] B_{sc}^2 B_{FC}^2 \phi_{sc} = 0 \quad (5)$$

$$\nabla^4 \phi_{FC} - \left[B_{sc}^2 + B_{FC}^2\right] \nabla^2 \phi_{FC} + \left[1 - k_c\right] B_{sc}^2 B_{FC}^2 \phi_{FC} = 0 \quad (6)$$

where $B_{FC}^2 = \frac{1}{\tau_c} = \frac{\Sigma_{FC}}{D_{FC}}$ (See Glasstone and Edlund, p. 228, 241) (7)

$$B_{sc}^2 = \frac{\Sigma_{sc}}{D_{sc}} \quad (8)$$

Equations (5) and (6) may be factored as follows:

$$(\nabla^2 - B_{rc}^2)(\nabla^2 + B_{ic}^2) \phi_{sc} = 0 \quad (9)$$

$$(\nabla^2 - B_{rc}^2)(\nabla^2 + B_{ic}^2) \phi_{FC} = 0 \quad (10)$$

where $B_{rc}^2 = R + \frac{B_{sc}^2 + B_{FC}^2}{2}$ (11)

$$B_{ic}^2 = R - \frac{B_{sc}^2 + B_{FC}^2}{2} \quad (12)$$

$$R = \sqrt{\left(\frac{B_{sc}^2 + B_{FC}^2}{2}\right)^2 + (k_c - 1) B_{FC}^2 B_{sc}^2} \quad (13)$$

Solutions of the four wave equations:

$$\nabla^2 \phi_{FC} - B_{rc}^2 \phi_{FC} = 0 \tag{14}$$

$$\nabla^2 \phi_{sc} - B_{rc}^2 \phi_{sc} = 0 \tag{15}$$

$$\nabla^2 \phi_{FC} + B_{ic}^2 \phi_{FC} = 0 \tag{16}$$

$$\nabla^2 \phi_{sc} + B_{ic}^2 \phi_{sc} = 0 \tag{17}$$

are solutions of the original equations (1) and (2). Thus, the sum of solutions to (14) and (16) is a solution of (2) and the sum of solutions to (15) and (17) is a solution of (1). We note that the solutions have the form:

$$\phi_{sc} = \frac{A}{r} \sin B_{ic}r + \frac{A'}{r} \cos B_{ic}r + \frac{B}{r} \sinh B_{rc}r + \frac{B'}{r} \cosh B_{rc}r \tag{18}$$

$$\phi_{FC} = \frac{A''}{r} \sin B_{ic}r + \frac{A'''}{r} \cos B_{ic}r + \frac{B''}{r} \sinh B_{rc}r + \frac{B'''}{r} \cosh B_{rc}r \tag{19}$$

As r approaches zero, terms containing cosh and cos become infinite while the others approach finite limits. Since by the boundary condition (5) stated in the introduction to the derivation, the fluxes are everywhere finite, the coefficients A', B', A''', and B''' must be identically zero. We have then:

$$\phi_{sc} = \frac{A}{r} \sin B_{ic}r + \frac{B}{r} \sinh B_{rc}r \tag{20}$$

$$\phi_{FC} = \frac{A''}{r} \sin B_{ic}r + \frac{B''}{r} \sinh B_{rc}r \tag{21}$$

Only two of the constants A, B, A'', B'' are independent. Since permissible solutions of (2) are $\frac{A}{r} \sin B_{ic}r$ for ϕ_{sc} and $\frac{A''}{r} \sin B_{ic}r$ for ϕ_{FC} we find by substitution in (2) that

$$\alpha_{ic} = \frac{A''}{A} = \frac{D_{sc}}{D_{FC}} \left[\frac{B_{sc}^2 + B_{ic}^2}{B_{FC}^2} \right] \tag{22}$$

and in similar fashion

$$\alpha_{rc} = \frac{B''}{B} = \frac{D_{sc}}{D_{FC}} \left[\frac{B_{sc}^2 - B_{rc}^2}{B_{FC}^2} \right] \tag{23}$$

B. Blanket equations

The blanket equations are

$$\nabla^2 \phi_{FB} - B_{FB}^2 \phi_{FB} + \frac{k_B}{p_B} \frac{\Sigma_{SB}}{D_{FB}} \phi_{SB} = 0 \qquad (24)$$

$$\nabla^2 \phi_{SB} - B_{SB}^2 \phi_{SB} + \frac{D_{FB}}{D_{SB}} \kappa_{RB}^2 \phi_{FB} = 0 \qquad (25)$$

Note that the resonance escape probability is not neglected in the blanket.

$$B_{FB}^2 = \kappa^2_{FB} + \kappa^2_{RB} \qquad (26)$$

$$\kappa^2_{FB} = \frac{1}{\tau_B} = \frac{\Sigma_{FB}}{D_{FB}} \qquad (27)$$

$$\kappa^2_{RB} = \frac{\Sigma_{RB}}{D_{FB}} \qquad (28)$$

$$B_{SB}^2 = \frac{\Sigma_{SB}}{D_{FB}} \qquad (29)$$

In obtaining solutions to (24) and (25) the procedure is similar to that used for the core equations. Two fourth order differential equations are formed by substituting ϕ_{FB} from (25) into (24), and by substituting ϕ_{SB} from (24) into (25). These are solved in the same manner as the core equations to yield:

$$\phi_{SB} = \frac{E'}{r} \sinh B_{iB}r + \frac{E''}{r} \cosh B_{iB}r + \frac{G'}{r} \sinh B_{rB}r + \frac{G''}{r} \cosh B_{rB}r \qquad (30)$$

$$\phi_{FB} = \frac{E'''}{r} \sinh B_{iB}r + \frac{E''''}{r} \cosh B_{iB}r + \frac{G'''}{r} \sinh B_{rB}r + \frac{G''''}{r} \cosh B_{rB}r \qquad (31)$$

where

$$B_{iB}^2 = \left[\frac{B_{FB}^2 + B_{SB}^2}{2}\right] - S \tag{32}$$

$$B_{rB}^2 = \left[\frac{B_{FB}^2 + B_{SB}^2}{2}\right] + S \tag{33}$$

$$S = \sqrt{\frac{k_B}{p_B} \chi^2_{FB} B_{SB}^2 - B_{FB}^2 B_{SB}^2 + \left[\frac{B_{FB}^2 + B_{SB}^2}{2}\right]^2} \tag{34}$$

Note that since $k_B/p_B < 1$ and, by (26), $B_{FB}^2 > \chi^2_{FB}$, S is always less than $\left[(B_{FB}^2 + B_{SB}^2)/2\right]$.

We note now that the fast and slow fluxes vanish at the extrapolated radius of the reactor. Since this radius for a large reactor is very close to the actual radius, little is sacrificed in accuracy and a considerable saving of labor is achieved by saying that the flux vanishes at the actual outside radius. Hence

$$\phi_{SB}(r = a_2) = \frac{E'}{a_2} \sinh B_{iB} a_2 + \frac{E''}{a_2} \cosh B_{iB} a_2 + \frac{G'}{a_2} \sinh B_{rB} a_2 + \frac{G''}{a_2} \cosh B_{rB} a_2 = 0 \tag{35}$$

$$\phi_{FB}(r = a_2) = \frac{E'''}{a_2} \sinh B_{iB} a_2 + \frac{E''''}{a_2} \cosh B_{iB} a_2 + \frac{G'''}{a_2} \sinh B_{rB} a_2 + \frac{G''''}{a_2} \cosh B_{rB} a_2 = 0 \tag{36}$$

We now establish the coupling coefficients between E' and E''', E'' and E'''', G' and G'''. G'' and G''''. We know that permissible solutions of (24) and (25) are $\frac{E'}{r} \sinh B_{iB}r$ and $\frac{E'''}{r} \sinh B_{iB}r$ for slow and fast fluxes respectively. Since $\nabla^2 \phi_{SB} = B_{iB}^2 \phi_{SB}$, (a result of the factoring process upon the fourth order equation in ϕ_{FB}), (25) becomes:

$$B_{iB}^2 \; \frac{E'}{r} \sinh B_{iB}r - B_{SB}^2 \; \frac{E'}{r} \; \sinh B_{iB}r + \left(\frac{D_{FB}\, \mathcal{K}^2{}_{FB}}{D_{SB}}\right) \; E''' \; \sinh B_{iB}r \; = 0 \tag{37}$$

$$E''' \; = \; \frac{D_{SB}}{D_{FB}} \left(\frac{B_{SB}^2 - B_{iB}^2}{\mathcal{K}^2{}_{FB}}\right) \; E' = \alpha_{iB}E' \tag{38}$$

Similarly we obtain

$$E'''' = E'' \; \frac{D_{SB}}{D_{FB}} \left(\frac{B_{SB}^2 - B_{iB}^2}{\mathcal{K}^2{}_{FB}}\right) = \alpha_{iB}E'' \tag{39}$$

$$G''' \; = \left(\frac{B_{SB}^2 - B_{rB}^2}{\mathcal{K}^2{}_{FB}}\right) \frac{D_{SB}}{D_{FB}} \; G' = \alpha_{rB} \; G' \tag{40}$$

$$G'''' = \frac{D_{SB}}{D_{FB}} \left(\frac{B_{SB}^2 - B_{rB}^2}{\mathcal{K}^2{}_{FB}}\right) G'' = \alpha_{rB} \; G'' \tag{41}$$

Now let

$$E' \; = -E \cosh B_{iB}J \tag{42}$$

$$E'' = \; E \sinh B_{iB}J \tag{43}$$

$$G' \; = -G \cosh B_{rB}K \tag{44}$$

$$G'' = \; G \sinh B_{rB}K \tag{45}$$

Then

$$\phi_{SB} = \frac{E}{r} \sinh B_{iB} \; (J - r) + \frac{G}{r} \sinh B_{rB} \; (K - r) \tag{46}$$

$$\phi_{FB} = \alpha_{iB} \frac{E}{r} \sinh B_{iB} \; (J - r) + \alpha_{rB} \frac{G}{r} \sinh B_{rB} \; (K - r) \tag{47}$$

We know that $\phi_{SB} = \phi_{FB} = 0$ at $r = a_2$. Therefore

$$E \sinh B_{iB} \; (J - a_2) = \; - G \sinh B_{rB} \; (K - a_2) \tag{48}$$

$$\alpha_{iB} \; E \sinh B_{iB} \; (J - a_2) = \alpha_{rB} \; G \sinh B_{rB} \; (K - a_2) \tag{49}$$

$$\alpha_{iB} \; E \sinh B_{iB} \; (J - a_2) = \alpha_{rB} \; E \sinh B_{iB} \; (J - a_2) \tag{50}$$

Since α_{iB} and α_{rB} are necessarily unequal, $J = a_2$ if E is not zero.

If E were zero, it follows that either G is zero (a trivial solution) or $K = a_2$. In general both G and E will be unequal to zero. Hence $J = K = a_2$ and we have

$$\phi_{SB} = \frac{E}{r} \sinh B_{iB} (a_2 - r) + \frac{G}{r} \sinh B_{rB} (a_2 - r) \tag{51}$$

$$\phi_{FB} = \frac{\alpha_{iB}}{r} E \sinh B_{iB} (a_2 - r) + \frac{\alpha_{rB}\, G}{r} \sinh B_{rB} (a_2 - r) \tag{52}$$

3. <u>Application of Boundary Conditions at the Interface Between Regions</u>

Since we assume continuity of the fast and slow fluxes at $r = a_1$ we obtain the two equations

$$A\, S_{ic} + C\, S_{rc} - E\, S_{iB} - G\, S_{rB} = 0 \tag{53}$$

$$\alpha_{ic}\, A\, S_{ic} + \alpha_{rc}\, C\, S_{rc} - \alpha_{iB}\, E\, S_{iB} - \alpha_{rB}\, G\, S_{rB} = 0 \tag{54}$$

We assume next that the difference between the slow current in the blanket and the slow current in the core at $r = a_1$ is equal to the number of neutrons absorbed per square centimeter of shell surface per second:

$$- D_{SC} \left(\frac{d\phi_{SC}}{dr}\right)_{r = a_1} = - D_{SB} \left(\frac{d\phi_{SB}}{dr}\right)_{r = a_1} + \Sigma_{sa}\, t'\ \phi_{SC} \tag{55}$$

This leads to the equation

$$D_{SC}\, A \left\{ C_{ic} + \beta S_{ic} \right\} + D_{SC}\, C\, (C_{rc} + \beta S_{rc}) - D_{SB}\, (E\, C_{iB} + G\, C_{rB}) = 0 \tag{56}$$

We next say that the fast currents in core and blanket are equal at $r = a_1$. This yields a fourth equation

$$D_{FC}(\alpha_{ic}\, A\, C_{ic} + C\, \alpha_{rc}\, C_{rc}) - D_{FB}(E\, \alpha_{iB}\, C_{iB} + G\, \alpha_{rB}\, C_{rB}) = 0 \tag{57}$$

If we have specified the compositions and dimensions of the core and the blanket we have in equations (53), (54), (56), and (57) a system of four homogeneous equations in A, C, E, and G. To have a consistent set of solutions for these unknowns, the determinant of the coefficients must be zero. This is the critical equation for the reactor. If we set the determinant equal

to zero and solve for β, we obtain after some algebraic manipulations:

$$\beta = \frac{X}{Y + Z} \tag{58}$$

This enables us to get the shell thickness which will allow the reactor to be just critical.

Dividing (53), (54), and (57) by A gives the system

$$S_{rc} \frac{C}{A} - S_{iB} \frac{E}{A} - S_{rB} \frac{G}{A} = - S_{ic} \tag{59}$$

$$\alpha_{rc} S_{rc} \frac{C}{A} - \alpha_{iB} S_{iB} \frac{E}{A} - \alpha_{rB} S_{rB} \frac{G}{A} = -\alpha_{ic} S_{ic} \tag{60}$$

$$\alpha_{rc} C_{rc} \frac{C}{A} - \alpha_{iB} \sigma_F C_{iB} \frac{E}{A} - \alpha_{rB} \sigma_F C_{rB} \frac{G}{A} = -\alpha_{ic} C_{ic} \tag{61}$$

$$\frac{C}{A} = \frac{\begin{vmatrix} -S_{ic} & -S_{iB} & -S_{rB} \\ -\alpha_{ic} S_{ic} & -\alpha_{iB} S_{iB} & -\alpha_{rB} S_{rB} \\ -\alpha_{ic} C_{ic} & -\alpha_{iB} \sigma_F C_{iB} & -\alpha_{rB} \sigma_F C_{rB} \end{vmatrix}}{\begin{vmatrix} S_{rc} & -S_{iB} & -S_{rB} \\ \alpha_{rc} S_{rc} & -\alpha_{iB} S_{iB} & -\alpha_{rB} S_{rB} \\ \alpha_{rc} C_{rc} & -\alpha_{iB} \sigma_F C_{iB} & -\alpha_{rB} \sigma_F C_{rB} \end{vmatrix}} \tag{62}$$

$$\frac{C}{A} \quad \frac{-S_{ic} S_{iB} S_{rB}}{S_{rc} S_{iB} S_{rB}} = \frac{\begin{vmatrix} 1 & 1 & 1 \\ \alpha_{ic} & \alpha_{iB} & \alpha_{rB} \\ \alpha_{ic} U_{ic} & \alpha_{iB} \sigma_F U_{iB} & \alpha_{rB} \sigma_F U_{rB} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ \alpha_{rc} & \alpha_{iB} & \alpha_{rB} \\ \alpha_{rc} U_{rc} & \alpha_{iB} \sigma_F U_{iB} & \alpha_{rB} \sigma_F U_{rB} \end{vmatrix}} \tag{63}$$

$$\frac{C}{A} = \frac{S_{ic}}{S_{rc}} \frac{\begin{vmatrix} 0 & 1 & 0 \\ \alpha_{ic} - \alpha_{iB} & \alpha_{iB} & \alpha_{rB} - \alpha_{iB} \\ \alpha_{ic} U_{ic} - \alpha_{iB} \sigma_F U_{iB} & \alpha_{iB} \sigma_F U_{iB} & \alpha_{rB} \sigma_F U_{rB} - \alpha_{iB} \sigma_F U_{iB} \end{vmatrix}}{\begin{vmatrix} 0 & 0 & 1 \\ \alpha_{rc} - \alpha_{rB} & \alpha_{iB} - \alpha_{rB} & \alpha_{rB} \\ \alpha_{rc} U_{rc} - \alpha_{rB} \sigma_F U_{rB} & \alpha_{iB} \sigma_F U_{iB} - \sigma_F \alpha_{rB} U_{rB} & \sigma_F \alpha_{rB} U_{rB} \end{vmatrix}} \tag{64}$$

$$\frac{C}{A} = \frac{S_{ic}}{S_{rc}} \frac{(cg - ai)}{(bi - ch)} = \frac{S_{ic}}{S_{rc}} \frac{Z}{Y} \tag{65}$$

In similar fashion:

$$\frac{E}{A} = \frac{S_{ic}}{S_{iB}} \frac{M}{Y} \tag{66}$$

$$\frac{G}{A} = \frac{S_{ic}}{S_{rB}} \frac{N}{Y} \tag{67}$$

Calculation Procedure

1. Nuclear Constants

The nuclear constants used were the same as those cited by Visner in CF-51-10-110. Material constants derived from these were calculated by the methods described therein. For the present purposes, it was assumed that there were 3% voids in the core. It was also assumed that whatever the blanket material considered, it would be in a sufficiently dispersed form to allow the blanket to be thought of as homogeneous.

The constants used for the various blanket materials are listed in the following table: The assumed temperature was 250°C. The moderator was heavy water.

	1000 G Th/1 (slurry)	4218 g Th/1 (ThO_2 pellets)	7000 g Th/1 (Th metal balls)
D_{SB}, cm.	1.152	1.01535	0.984568
D_{FB}, cm.	1.429	1.05018	0.966684
$\sum_{RB}$, cm.$^{-1}$	4.619×10^{-3}	14.11516×10^{-3}	20.9612×10^{-3}
τ_B, cm.^{2}	186.8	256.508	274.66
$\sum_{SB}$, cm.$^{-1}$	13.526×10^{-3}	56.95894×10^{-3}	94.4914×10^{-3}

The large number of significant figures shown were employed only to insure a high degree of consistency in the calculations, and are not intended to convey any false impressions concerning the accuracy of the constants.

The stainless steel shell was estimated to have a macroscopic cross section of 0.183 cm.$^{-1}$ at 250°C.

The ThO_2 pellets were assumed to have a density of 8 g/cm^3 and that the blanket was composed of 60% ThO_2 and 40% heavy water by volume. Th metal density was taken as 11.664 g/cm^3 and the metal balls were also assumed to occupy 60% of the blanket volume.

2. Procedure

Typical calculation sheets are displayed on p. 51 ff. of CF-51-10-110. We find a value of β (equivalent to finding a shell thickness) such that the determinant of the coefficients of equations (53), (54), (56) and (57) is zero. This value of β is given by equation (67). We repeat the calculations until we find the desired shell thickness, changing the core concentration in the process. For example, in our work we wished to find the critical core concentrations corresponding to shell thicknesses of 1/8 inch and 1/4 inch. Several core concentrations were guessed, and the corresponding shell thicknesses were computed. Then by extrapolation or interpolation the value of the core concentrations corresponding to 1/8 inch and 1/4 inch

thicknesses were obtained.

At this point, the calculations can be subjected to two checks: equivalence of fast fluxes in core and blanket at a_1, and equivalence of slow fluxes in core and blanket at a_1. These are obtained from the equations:

$$Y = M + N - Z \text{ (slow flux)} \tag{68}$$

$$(C_{rc} + \beta S_{rc})\, C - \sigma_s\, (C_{iB}\, E + C_{rB} G) = -C_{ic} - \beta S_{ic} \text{ (fast flux)} \tag{69}$$

To calculate the neutron balance, the reactor is said to operate at that power at which $A \equiv 1$. At this power we calculate the volume integrals of fast and slow flux in core and blanket. These are multiplied by the macroscopic cross sections of the materials in the reactor regions to give numbers of neutrons absorbed per second per cm^3 in U^{235}, U^{238}, D_2O, Th, etc. Having these numbers, the leakage rate and shell absorption figures, we can calculate how many neutrons are absorbed or leaked in any fashion per neutron absorbed in an atom of U^{235}. The sum of all these numbers gives a final check, for the total number of neutrons absorbed or leaked in any way per neutron absorbed in U^{235} must equal the number of neutrons produced by fission per absorption of a neutron in U^{235}. This last is simply

$$\eta = \nu \frac{\Sigma_f\ (25)}{\Sigma_a\ (25)} \tag{70}$$

where $\nu = 2.5$ and the cross-sections are thermal.

This last check is valid, provided the reactor is thermal, as is assumed in the calculation of k_c.

The necessary formulas for calculating the neutron balance are easily derived:

1. Shell absorption. (It is assumed that the shell captures only slow neutrons.)

$$\phi_{sc}\ (a_1) = \frac{A}{a_1}\ \sin B_{ic}\ a_1 + \frac{C}{a_1}\ \sinh B_{rc}\ a_1 \qquad (71)$$

$$= \frac{A}{a_1}\ S_{ic} + \frac{C}{a_1}\ S_{rc}$$

For $A \equiv 1$,

$$\text{Shell absorption} = 4\pi\ a_1^{\ 2}\ \Sigma_{sa}\ t'\ \phi_{sc}\ (a_1) \qquad (72)$$

$$= 4\pi\ \beta D_{sc}\ \left[S_{ic} + C\ S_{rc}\right] \qquad (73)$$

Since the 4π cancels out when the balance is calculated, it was dropped in the calculation sheets of CF-51-10-110.

2. Leakage of fast and slow neutrons.

For the slow flux:

$$\text{Positive current at } a_2 = \frac{\phi_{sB}}{4} - \frac{D_{sB}}{2}\ \frac{d\phi_{sB}}{dr} \qquad (74)$$

$$\text{Negative current at } a_2 = \frac{\phi_{sB}}{4} + \frac{D_{sB}}{2}\ \frac{d\phi_{sB}}{dr} \qquad (75)$$

It is assumed that no neutrons which leave the reactor ever return

Combining (74) and (75):

$$\text{Positive current of slow neutrons at } a_2 = \frac{\phi_{sB}\ (r = a_2)}{2}.$$

$$\text{Total slow leakage} = 4\pi a_2^{\ 2}\ \phi_{sB}\ (r = a_2)/2 \qquad (76)$$

We cannot say here that $\phi_{sB}\ (a_2) = 0$, but we use the more exact formula for the flux:

$$\phi_{sB} = \frac{E}{r} \sinh B_{iB}\ (a_2 + E_s - r) + \frac{G}{r}\ \sinh B_{rB}\ (a_2 + E_s - r) \qquad (77)$$

Hence

$$\text{Slow leakage} = \frac{1}{2}\ a_2\ (E \sinh B_{iB}\ E_s + G \sinh B_{rB}\ E_s) \qquad (78)$$

In similar fashion we may obtain

Fast leakage

$$= \frac{1}{2} a_2 \left(E \alpha_{iB} \sinh B_{iB} E_f + \alpha_{rB} G \sinh B_{rB} E_f\right) \qquad (79)$$

Note that the 4π is dropped from (78) and (79) as it was from (73).

3. Integrated fluxes.

$$F_{sc} = - \frac{C_{ic}}{B_{ic}^2} + \frac{C\, C_{rc}}{B_{rc}^2} \qquad (80)$$

$$F_{FC} = - \alpha_{ic} \frac{C_{ic}}{B_{ic}^2} + \alpha_{rc} \frac{C\, C_{rc}}{B_{rc}^2} \qquad (81)$$

$$F_{SB} = - \frac{E}{B_{iB}^2} \left(B_{iB} a_2 + C_{iB}\right) - \frac{G}{B_{rB}^2} \left(B_{rB} a_2 + C_{rB}\right) \qquad (82)$$

$$F_{FB} = - \alpha_{iB} \frac{E}{B_{iB}^2} \left(B_{iB} a_2 + C_{iB}\right) - \frac{\alpha_{rB} G}{B_{rB}^2} \left(B_{rB} a_2 + C_{rB}\right) \qquad (83)$$

These were obtained merely by integrating the flux equations (20), (21), (51), and (52) over the proper reactor regions.

TABULATION OF RESULTS FOR REACTORS AT 250°C

Blanket Type		1000 g Th/l as ThO_2 Slurry in D_2O		4218 g Th/l as ThO_2 Pellets in D_2O		7000 g Th/l as Th Pellets in D_2O	
Blanket Thickness		2 ft.	2 ft.	2 ft.	2 ft.	2 ft.	2 ft.
Shell Thickness		1/8 in.	1/4 in.	1/8 in.	1/4 in.	1/8 in.	1/4 in.
Critical Concentration (g. U/liter)		4.470	4.785	5.15	5.30	5.430	5.549
<u>Absorption</u>							
Core	U^{235}	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
	U^{238}	0.000289	0.000289	0.00029	0.00029	0.000289	0.000288
	S	0.000821	0.000821	0.00082	0.00082	0.000821	0.000821
	D_2O	0.006988	0.00653	0.00598	0.00581	0.005753	0.005630
Blanket	Th(fast)	0.213483	0.20892	0.42327	0.41853	0.475370	0.471196
	Th(slow)	0.688540	0.58171	0.57756	0.50446	0.551862	0.487059
	D_2O	0.001377	0.00116	0.00014	0.00013	0.000083	0.000073
Shell		0.194282	0.30711	0.11258	0.19063	0.087046	0.156123
	Slow	0.009377	0.00867	0.00007	0.00007	0.000007	0.000007
Leakage	Fast	0.007415	0.00726	0.00069	0.00068	0.000134	0.000132
U^{233} atoms produced per U^{235} atom destroyed		0.9020	0.79063	1.00083	0.92299	1.027297	0.958255

NOTE: In view of uncertainties in the nuclear constants employed to arrive at the above figures, these numbers are accurate only to two significant figures regardless of the number displayed above.

TABLE OF NOMENCLATURE

1. ∇^2 = Laplacian operator. In spherical coordinates with no angular dependence equal to

$$\frac{d^2}{dr^2} + \frac{2}{r} \quad \frac{d}{dr}$$

2. r = Radius variable, cm.
3. ϕ_{FC} = Fast flux in the core, neutrons per cm^2 per second.
4. ϕ_{SC} = Slow flux in the core, neutrons per cm^2 per second.
5. ϕ_{SB} = Slow flux in the blanket, neutrons per cm^2 per second.
6. ϕ_{FB} = Fast flux in the blanket, neutrons per cm^2 per second.
7. D_{FC} = Fast diffusion coefficient in the core, cm.
8. D_{SC} = Slow diffusion coefficient in the core, cm.
9. D_{SB} = Slow diffusion coefficient in blanket, cm.
10. D_{FB} = Fast diffusion coefficient in blanket, cm.
11. τ_B = Fermi age in blanket, cm^2.
12. τ_C = Fermi age for core, cm^2.
13. Σ_{SC} = Macroscopic cross section for absorption of slow neutrons in the core, cm^{-1}.
14. Σ_{FC} = Macroscopic cross section for degradation of fast neutrons to slow neutrons in t
15. Σ_{FB} = Macroscopic cross section for degradation of fast neutrons to slow neutrons in the blanket, cm^{-1}.
16. Σ_{SB} = Macroscopic absorption cross section in the blanket for slow neutrons, cm^{-1}.
17. Σ_{RB} = Macroscopic absorption cross section in the blanket for fast neutrons, cm^{-1}.

18. k_c = Infinite multiplication constant in the core.

19. k_B = Infinite multiplication constant for the blanket.

20. p_B = Resonance escape probability in the blanket.

21. $B^2_{FC} = 1/\tau_C$

22. $B_{SC}^2 = \Sigma_{SC}/D_{SC}$

23. $R^2 = \left[(B^2_{SC} + B^2_{FC})/2\right]^2 + (k_C - 1)\, B^2_{SC}\, B^2_{FC}$

24. $B^2_{iC} = R - \left[(B^2_{SC} + B^2_{FC})/2\right]$

25. $B^2_{rC} = R + \left[(B^2_{SC} + B^2_{FC})/2\right]$

26. $\alpha_{iC} = D_{SC}(B_{SC}^2 + B_{iC}^2)/D_{FC}B^2_{FC}$

27. $\alpha_{rC} = D_{SC}(B_{SC}^2 - B_{rC}^2)/D_{FC}B_{FC}^2$

28. $\mathcal{K}^2_{FB} = 1/\tau_B$

29. $B_{SB}^2 = \Sigma_{SB}/D_{FB}$

30. $\mathcal{K}^2_{rB} = \Sigma_{RB}/D_{FB}$

31. $B^2_{FB} = \mathcal{K}^2_{FB} + \mathcal{K}^2_{RB}$

32. $S^2 = \frac{k_B}{p_B}\mathcal{K}^2_{FB}\, B^2_{SB} - B^2_{FB}\, B^2_{SB} + \left[(B^2_{FB} + B^2_{SB})/2\right]^2$

33. $B_{iB}^2 = \left[(B_{FB}^2 + B_{SB}^2)/2\right] - S$

34. $B_{rB}^2 = \left[(B_{FB}^2 + B_{SB}^2)/2\right] + S$

35. a_1 = Radius of core, cm.

36. a_2 = Radius of blanket, cm.

37. $t' = a_2 - a_1$

38. $\alpha_{iB} = D_{SB}(B_{SB}^2 - B_{iB}^2) / (D_{FB} \mathcal{H}_{FB}^2)$

39. $\alpha_{rB} = D_{SB}(B_{SB}^2 - B_{rB}^2) / (D_{FB} \mathcal{H}_{FB}^2)$

40. $S_{iC} = \sin B_{iC} a_1$

41. $S_{iB} = \sinh B_{iB} t'$

42. $T_{iC} = B_{iC} a_1 \cos B_{iC} a_1$

43. $C_{iC} = T_{iC} - S_{iC}$

44. $U_{iC} = C_{iC}/S_{iC}$

45. $T_{iB} = B_{iB} a_1 \cosh B_{iB} t'$

46. $C_{iB} = -T_{iB} - S_{iB}$

47. $U_{iB} = C_{iB}/S_{iB}$

48. $\sigma_S = D_{SB}/D_{SC}$

49. $S_{rC} = \sinh B_{rc} a_1$

50. $S_{rB} = \sinh B_{rB} t'$

51. $T_{rC} = B_{rC} a_1 \cosh B_{rC} a_1$

52. $C_{rC} = T_{rC} - S_{rC}$

53. $U_{rC} = C_{rC}/S_{rC}$

54. $T_{rB} = B_{rB}\, a_1 \cosh B_{rB}\, t'$

55. $C_{rB} = -T_{rB} - S_{rB}$

56. $U_{rB} = C_{rB}/S_{rB}$

57. $\sigma_F = D_{FB}/D_{FC}$

58. $a = \alpha_{iC} - \alpha_{iB}$

59. $b = \alpha_{rC} - \alpha_{rB}$

60. $c = \alpha_{rB} - \alpha_{iB}$

61. $d = U_{iC} - \sigma_S\, U_{iB}$

62. $e = U_{rC} - \sigma_S\, U_{rB}$

63. $f = \sigma_S(U_{rB} - U_{iB})$

64. $g = \alpha_{iC}\, U_{iC} - \sigma_F\, \alpha_{iB}\, U_{iB}$

65. $h = \alpha_{rC}\, U_{rC} - \sigma_F\, \alpha_{rB}\, U_{rB}$

66. $i = \sigma_F\, \alpha_{rB}\, U_{rB} - \sigma_F\, \alpha_{iB}\, U_{iB}$

67. $m = \alpha_{iC} - \alpha_{rB}$

68. $n = \alpha_{iC}\, U_{iC} - \sigma_F\, \alpha_{rB}\, U_{rB}$

69. $r = \alpha_{iB} - \alpha_{rC}$

70. $s = \sigma_F\, \alpha_{iB}\, U_{iB} - \alpha_{rC}\, U_{rC}$

71. $X = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

72. $Y = \begin{vmatrix} b & c \\ h & i \end{vmatrix}$

73. $Z = \begin{vmatrix} c & a \\ i & g \end{vmatrix}$

74. $M = \begin{vmatrix} m & h \\ n & b \end{vmatrix}$

75. $N = \begin{vmatrix} a & r \\ g & s \end{vmatrix}$

76. t = Core tank thickness, cm.

77. Σ_{sa} = Macroscopic absorption cross section for absorption of core tank wall, cm^{-1}.

78. $\beta = \dfrac{a_1 \Sigma_{sa} t}{D_{Sc}}$

79. A = A coefficient (see equation 18, p. 12) arbitrarily set equal to unity.

80. $C = \dfrac{S_{1C}}{S_{rC}} \quad \dfrac{Z}{Y}$

81. $G = \dfrac{S_{1C}}{S_{rB}} \quad \dfrac{N}{Y}$

82. $E = \dfrac{S_{1C}}{S_{1B}} \quad \dfrac{M}{Y}$

83. E_S = 2.13 D_{SB}, slow extrapolation distance, cm.

84. E_F = 2.13 D_{FB}, fast extrapolation distance, cm.

85. $4\pi F_{SC}$ = Volume integral of slow flux in core.

86. $4\pi F_{FC}$ = Volume integral of fast flux in core.

87. $4\pi F_{SB}$ = Volume integral of slow flux in blanket.

88. $4\pi F_{FB}$ = Volume integral of fast flux in blanket.